

# Fast radio bursts as giant pulses from young rapidly rotating pulsars

Maxim Lyutikov<sup>1</sup>, Lukasz Burzawa<sup>1</sup>, Sergei B. Popov<sup>2</sup>

<sup>1</sup> *Department of Physics, Purdue University, 525 Northwestern Avenue, West Lafayette, IN 47907-2036, USA; lyutikov@purdue.edu*

<sup>2</sup> *Sternberg Astronomical Institute, Lomonosov Moscow State University, Universitetsky prospekt 13, 119991, Moscow, Russia*

## ABSTRACT

We discuss possible association of fast radio bursts (FRBs) with supergiant pulses emitted by young pulsars (ages  $\sim$  tens to hundreds of years) born with regular magnetic field but very short – few milliseconds – spin periods. FRBs are extra-Galactic events coming from distances  $d \lesssim 100$  Mpc. Most of the dispersion measure (DM) comes from the material in the freshly ejected SNR shell; for a given burst the DM should decrease with time. FRBs are not expected to be seen below  $\sim 300$  MHz due to free-free absorption in the expanding ejecta. A supernova might have been detected years before the burst; FRBs are mostly associated with star forming galaxies.

The model requires that some pulsars are born with very fast spins, of the order of few milliseconds. The observed distribution of spin-down powers  $\dot{E}$  in young energetic pulsars is consistent with equal birth rate per decade of  $\dot{E}$ . Accepting this injection spectrum and scaling the intrinsic brightness of FRBs with  $\dot{E}$ , we predict the following properties of a large sample of FRBs: (i) the brightest observed events come from a broad distribution in distances; (ii) for repeating bursts brightness either remains nearly constant (if the spin-down time is longer than the age of the pulsar) or decreases with time otherwise; in the latter case  $DM \propto \dot{E}$ .

## 1. Introduction

Fast radio bursts (FRBs) (Lorimer et al. 2007; Keane et al. 2012; Thornton et al. 2013; Kulkarni et al. 2014; Spitler et al. 2014) are recently identified mysterious events that are still waiting to be understood. Let us first summarize briefly the main observation properties and the key inferences. (Though some of the inferences listed below are based on single FRBs we

assume that these are common properties. Also, see below a separate paragraph discussing the result of Keane et al. (2016).)

- FRBs are typically milliseconds long events. The observed duration is mostly due to scattering broadening during propagation (Champion et al. 2015); intrinsic width is consistent with the  $\delta$ -function emission in time. Still, the observed duration limits can be translated to limits on the scale of the emission regions as  $\leq 10^8$  cm in size. Relativistic bulk motion with  $\Gamma_b$  would modify this estimate by a factor  $\Gamma_b^2$ , but then the corresponding event rates would increase accordingly.
- The rates are estimated as  $\sim 10^3 - 10^4$  per day per sky above 4 mJy per msec (*e.g.* Rane et al. 2016, and references therein). The upper limit is barely comparable to the SN rate up to  $z \leq 1$  (*e.g.* Bazin et al. 2009), however, the short duration is inconsistent with the SN explosion. The high rate of FRBs also excludes violent events like NS-NS mergers (*e.g.* Falcke & Rezzolla 2014), that are expected to have rates at least hundred times lower (Phinney 1991; Burgay et al. 2003). Also, the observed rate of FRBs is, probably, just a lower limit – there can be an even more numerous population of weaker FRBs (as demonstrated by the repetitive FRB, Spitler et al. 2016)
- Dispersion measure (DM) of FRBs is in the range few hundreds to few thousands. If DM is due to the intergalactic medium this would place FRBs at cosmological distances,  $z \sim 1$ . However, DM is likely to come from the local structures near the sources (Masui et al. 2015). If DM is local, the isotropic distribution of FRBS on the sky (Petroff et al. 2014; Macquart & Johnston 2015) implies that the typical distance  $\geq$  few tens Mpc.
- FBRs are repetitive but non-periodic (Spitler et al. 2016); present-day overall limits on repeatability (Petroff et al. 2015b) corresponds to, approximately, not more than a burst per day for bright bursts (in terms of observed bursts; for highly beamed emission into an angle  $1/\Delta\Omega_b$  the intrinsic repetitiveness will be larger by  $4\pi/\Delta\Omega_b$ ). In case of weaker bursts the repetition rate can be higher (Spitler et al. 2016). Repetitiveness also excludes violent events like compact object mergers.
- Multi-component structures (Champion et al. 2015) hint at rotation at  $\sim$  millisecond periods. Multi-component structures can be used as an argument against catastrophic models like collapse of a NS to a BH, or deconfinement of matter and formation of a quark star (Champion et al. 2015).
- FBRs have relatively flat spectra with index at least flatter than -3.2 (Caleb et al. 2016), or even more flat (Rowlinson et al. 2016). On the other hand, spectra can be highly variable, possibly with narrow spectral component (Spitler et al. 2016).

- FRBs show both circular (Petroff et al. 2015a) and/or linear polarization (Masui et al. 2015) intrinsic to the source. In addition, Masui et al. (2015) detected intrinsic position angle (PA) rotation during the burst, possibly consistent with PA swings observed in pulsar (Radhakrishnan & Cooke 1969).

Many of the above properties stand in sharp contrast to the recently claimed identification of the FRB host with an elliptical galaxy at  $z = 0.45$  (the claimed DM would then be cosmological; the event would be consistent with energetic violent events like NS-NS merger, Keane et al. 2016). Williams & Berger (2016) (see also ATel #8752) claimed that very high persistent radio emission is inconsistent with the low star-formation rate – the host is probably an AGN. Also, small galactic latitude implies lots of scattering in the Galaxy; the time scale of few days is typical for AGN intra-day variability (Jauncey et al. 2001). From the theoretical point of view, the result of Keane et al. (2016) – the implied very bright afterglow, – contradicts the fact that the rate of FRBs is hundreds of times higher than of violent merger events like NS-NS mergers. Also, the afterglow implied by Keane et al. (2016) needs about  $10^{45}$  ergs emitted in radio, different by about two orders of magnitude from the 2004 flare from SGR 1806-20 produced total  $4 \times 10^{43}$  ergs (Gaensler et al. 2005). The SGR afterglow lasted longer, with peak flux about 200 times higher than Keane’s burst (50 mJy versus  $250 \mu\text{Jy}$ ) from a distance more than 100 times closer. If the results of Keane et al. (2016), are confirmed, that would imply two types of FRB progenitors (type I - repeating, type II - non-repeating). Below we then limit our discussion to the “type I” FRBs.

## 2. FRB emission site: magnetospheres of neutron stars

Given the above inferences let us estimate parameters at the source. The key unknown is the distance. Given that the DM is local (Masui et al. 2015), it cannot be used as a distance estimate. Yet, isotropy of the observed events argues for a local cosmological origin. As a fiducial value we use a typical distance of  $d = 100$  Mpc and take Lorimer burst (Lorimer et al. 2007, duration 5 msec, peak flux  $S_{30\text{Jy}}$ , DM=375) as a prototypical example.

The instantaneous (isotropic-equivalent) luminosity  $L_{FRB}$  is then

$$L_{FRB} = 4\pi d^2(\nu F_\nu) = 3.4 \times 10^{41} S_{30\text{Jy}} d_{100\text{Mpc}}^2 \text{erg s}^{-1}, \quad (1)$$

while the total radiated energy is

$$E_{tot} = 4\pi d^2(\nu S_\nu)\tau = 1.7 \times 10^{39} d_{100}^2 \tau_{5\text{msec}} \nu_9 S_{30\text{Jy}} \text{erg} \quad (2)$$

Where  $\tau$  is the burst duration (we normalize it to 5 msec) and  $\nu$  is the observation frequency (normalized to 1 GHz). The energy density at the source corresponding to (2) is

$$u_{rad} \sim \frac{E_{tot}}{(c\tau)^3} = 5 \times 10^{14} \text{ erg cm}^{-3} \quad (3)$$

The brightness temperature

$$T_b \approx \frac{2\pi d^2 S_\nu}{\nu^2 \tau^2} \frac{\Delta\Omega}{4\pi} \approx 5 \times 10^{34} \text{ K} \quad (4)$$

clearly implies a coherent mechanism.

Radiation mechanism is likely to include particles in magnetic field. The magnetic field energy density corresponding to (3) is

$$B_{eq} = \sqrt{8\pi u_{rad}} = \sqrt{8\pi} \frac{\sqrt{L}}{c^{3/2}\tau} = 10^8 \text{ G}. \quad (5)$$

Another requirement for high magnetic field in the emission region comes from the estimate of the wave intensity parameter

$$a = \frac{eE}{m_e c \omega} \approx 10^5 \gg 1 \quad (6)$$

where  $E = \sqrt{L/(c^3\tau^2)}$  is the typical electric field in the wave at the emission site (Luan & Goldreich 2014). Since the emission is coherent, in unmagnetized plasma the emitting particles would have highly relativistic Lorentz factor  $\gamma_\perp \sim a$  and would quickly lose energy through various radiative processes (*e.g.* synchrotron in case of large momentum perpendicular to the magnetic field). In highly magnetized plasma (in the limit  $\omega \ll \omega_B$ ) the large value of the intensity parameter (6) does not necessarily imply high radiative losses of emitting particles. Instead of oscillation under the influence of the electric field of the wave with Lorentz factor  $\gamma \sim a$ , in a high magnetic field particles experience  $E \times B$  drift; we need then to replace in Eq. (6)  $\omega \rightarrow \omega_B$ ; using (15) we find  $a \sim 1/\sqrt{8\pi}$ . (Also, as is the case for pulsar radio emission, the radiation should escape induced Compton scattering in the wind (Wilson & Rees 1978); this can be achieved by sufficiently fast and/or rarefied wind (Sincell & Krolik 1992).

The above estimates, by exclusion, leave only magnetospheres of neutron stars as viable *loci* of the generation of FRBs (Popov & Postnov 2010; Pen & Connor 2015; Cordes & Wasserman 2016; Spitler et al. 2016)

## 2.1. Two possible mechanisms for FRBs

### 2.1.1. *Magnetically and rotationally powered: magnetars versus “pulsars on steroids”*

Identification of FRBs with neutron star and evidence against catastrophic events (collapse, coalescence, etc.) leave two possible mechanisms: (i) radio emission accompanying giant flares in magnetars (Lyutikov 2002; Popov & Postnov 2010; Lyubarsky 2014; Keane et al. 2012; Pen & Connor 2015); (ii) Giant pulses (GPs) analogues emitted by young pulsars (Lundgren et al. 1995; Soglasnov et al. 2004; Popov & Stappers 2007), as discussed by Cordes & Wasserman (2016); Connor et al. (2016b). Importantly, both scenarios imply repetitiveness and FRBs relation to young neutron stars with particular properties – high magnetic fields in the case of magnetars, or high spin-down energies in the case of GPs. These two possibilities rely on different source of energies for FRBs: strong magnetic fields in case of magnetars and rotational energy in case of GPs.

Comparing properties of FRBs with the radio emission of magnetars and/or GPs can, in principle, be used to favor one of the models, magnetically or rotationally powered, as we discuss next. However, lack of understanding of mechanisms of radio emission from neutron stars is a major impediment to this possibility (*e.g.* Melrose 1995; Lyutikov et al. 1999; Melrose & Gedalin 1999; Beskin et al. 2015). Note, that coherent curvature emission by bunches is not considered a viable emission mechanism (Melrose 1992; Melrose & Gedalin 1999).

### 2.1.2. *Different radio emission mechanisms*

Let us briefly outline our current understanding of the radio emission from neutron stars, a long-standing problem in astrophysics. Qualitatively we can identify *three* types/mechanisms of radio emission in neutron stars: (i) normal pulses, exemplified by Crab precursor (coming from opened field lines, probably near the polar cap, having log-normal distribution in fluxes), see Moffett & Hankins (1996); (ii) GPs, exemplified by Crab Main Pulses and Interpulses (coming from outer magnetosphere, near the last closed field lines; having power-law distribution in fluxes (Lundgren et al. 1995); possibly with a special subset of supergiant pulses, see Mickaliger et al. (2012); sometimes GPs show narrow spectral structure, see Hankins & Eilek (2007); Lyutikov (2007); (iii) radio emission from magnetars (coming from the region of close field lines, variable on secular times scales and having very flat spectra), *e.g.* Camilo et al. (2006).

Though comparison of these general properties of pulsar radio emission with FRBs is

surely inconclusive, we favor the Giant Pulses model for the following reasons. (i) Similar time scales of GPs and FRBs, when allowed for propagation broadening Champion et al. (2015). (ii) Polarization: similar to FRBs (Petroff et al. 2015a; Masui et al. 2015), GPs have strong polarization signals (Soglasnov 2007), often switching between linear and circular polarization. (iii) Association of FRBs with rotationally powered GPs allows testifiable predictions to be made, based on the possible scaling of the emitted intensity with the spin-down power. (iv) non-detection of radio emission during SGR 1806-20 giant flare (Tendulkar et al. 2016) provides arguments against the magnetar association. (However, note that Tendulkar et al. 2016, searched only for simultaneous  $\gamma$ - and radio signals. If there is a delay between them, then the argument is not applicable. Also, comparison with just one burst of one SGR can be not very constraining for the whole population.) (v) Though the inferred flatter spectra of FRBs (Keane et al. 2012) make them resemble magnetar radio emission (Camilo et al. 2006), below we argue that this can be explained by the low frequency free-free absorption. (vi) Possible narrow spectral features in FRBs (Spitler et al. 2016) resemble those seen in Crab GPs (Hankins & Eilek 2007).

### 3. The working model: giant pulses from young energetic pulsars

In the following we further discuss the possibility that FRBs are (super)-giant pulses from energetic newborn pulsars.

#### 3.1. DM, RM and free-free absorption from SNR

Given that the DM comes from the local environment (Masui et al. 2015) and that FRBs are related to neutron stars, how the values of  $DM \sim$  hundreds can be achieved? Both Galactic and intergalactic contributions to the DM from the distances of  $\leq 100$  Mpc is expected to be typically  $\sim$  tens. (High values of DM for some Galactic pulsars (Manchester et al. 2005) is due to our location in the plane of the Galaxy and the fact that many pulsars are located within the Galactic plane.) Typical values of DM of Galactic pulsars is  $\sim$  tens or hundreds (but normally below 375 – the lowest DM for FRBs). Thus, FRBs cannot come from a general extragalactic pulsar population, but should come from a special sub-class.

A possible alternative, that we adopt as the main model, is that the DM comes from a young SNR ejecta, i.e. from a dense shell around a newborn NS. Let us estimate the required time scales. Let's assume that a recent SN ejecta expelled mass  $M_{ej}$ . If the size of the SN

ejecta is  $r$ , the corresponding DM is

$$\text{DM} \approx \frac{M_{ej}}{m_p r^2} \quad (7)$$

So, the larger is the size, the smaller is the DM. For a given DM the size is

$$r = \sqrt{M_{ej}/m_p} \frac{1}{\sqrt{\text{DM}}} = 0.34 \text{pc} \sqrt{m_\odot} \text{DM}_{375}^{-1/2}, \quad (8)$$

where  $\text{DM}_{375} = \text{DM}/375$  and  $m_\odot = M_{ej}/M_\odot$ .

Swept-up mass

$$\frac{M_{swept}}{M_{ej}} = \sqrt{M_{ej}/m_p} \frac{n_{ISM}}{\text{DM}^{3/2} \text{pc}^{3/2}} = 4.5 \times 10^{-4} n_{ISM} \sqrt{m_\odot} \ll 1, \quad (9)$$

where  $n_{ISM}$  is the circumburst ISM number density. So the motion is typically ballistic with velocity

$$v_{ej} = \sqrt{\frac{2E_{ej}}{M_{ej}}}. \quad (10)$$

To reach the size (8) it takes

$$t = \frac{M_{ej}}{\sqrt{2\text{DM}E_{ej}m_p}} = 35 \text{yrs } m_\odot \quad (11)$$

(for  $E_{ej} = 10^{51}$  erg.)

Masui et al. (2015) claimed  $\text{RM} = 180 \text{ rad/m}^2$  and  $\text{DM} = 600 \text{ pc cm}^{-3}$  in the circumburst surrounding; this implies the average magnetic field

$$B = 2\pi \frac{m_e^2 c^4}{e^3} \frac{\text{RM}}{\text{DM}} = 3 \times 10^{-7} \text{G}, \quad (12)$$

below the typical Galactic field of  $\mu\text{G}$ . One possible explanation is that the magnetic field that produces the RM is confined to the expanding envelope – it is then expected to be in toroidal direction, perpendicular to the line of sight.

The free-free optical depth through an expanding SN shell is sufficiently small at  $\sim \text{GHz}$  frequencies (Lang 1999, Eq. 1.223)

$$\tau = 8 \times 10^{-2} n^2 \nu^{-2.1} r T^{-1.35} = 0.05 \text{DM}_{375}^{5/2} m_\odot^{-1/2} \nu_9^{-2.1}. \quad (13)$$

Note that the free-free optical depth becomes of the order of unity at frequencies  $\leq 300 \text{ MHz}$ . This might explain the fact that low frequency observatories like LOFAR and MWA did not

see FRBs (Karastergiou et al. 2015; Rowlinson et al. 2016), and that the lowest frequency of FRB detection so far is 700MHz (Masui et al. 2015; Connor et al. 2016a). The plasma frequency in the ejecta is  $\omega_p = \sqrt{4\pi n e^2 / m_e} = 10^6 \text{DM}_{375}^{3/4} (M_{ej}/M_\odot)^{-1/4} \text{ rad s}^{-1}$ , so the source is transparent to radio waves at  $\sim 1 \text{ GHz}$ .

We conclude that very young SNRs, at ages tens to hundreds of years can provide the DM of the order of the observed values.

### 3.2. Pulsar physics

Above we have established that the observed properties of FRBs are consistent with SN environment  $\sim$  tens of years after the explosions. Next, let us discuss how pulsar physics fits with these estimates. We hypothesize that FRBs are rare (super)giant pulses-like events whose luminosity  $L_{FRB}$  scales with the spin-down power of a pulsar,  $L_{FRB} = \eta \dot{E}$ ,  $\eta \ll 1$  (we note that this is not the case for the bulk of the pulsar population Manchester et al. 2005).

For Crab pulsar the peak GP fluxes  $S_\nu$  exceed Mega-Janskys (Hankins et al. 2003; Soglasnov 2007; Mickaliger et al. 2012). The corresponding *instantaneous* efficiency

$$\eta = \frac{L_{GP}}{\dot{E}_{Crab}} = \frac{\nu c^3 d_{Crab}^2 S_\nu P_{NS}^4}{4\pi^3 B_{NS}^2 R_{NS}^6} \approx 10^{-2}, \quad (14)$$

where subscript NS refers to the magnetic field on the surface, radius and period of Crab pulsar. Since the GP duration is much smaller than the period, the average efficiency is much smaller than (14) by  $\Delta\Omega/4\pi = (\Delta\theta)^2/4 \approx \text{few} \times 10^{-7}$  where  $\Delta\theta \approx 2\pi\tau_{GP}/P_{NS} \approx 10^{-3}$  is the relative active phase of the GP,  $\tau_{GP} \sim \text{few } \mu\text{sec}$  is the GP duration. Thus, instantaneously, pulsars GP luminosity can reach  $\eta \sim \text{few percent}$  of the spin down power.

By analogy with Crab GPs we expect that the intrinsic FRB duration is smaller than the neutron star spin. (FRB duration is consistent with  $\delta$ -function pulse smeared by propagation effects, Champion et al. 2015.) Normalizing the FRB duration to the neutron star spin, the required magnetic field is

$$L_{FRB} = \eta \dot{E} \rightarrow B_{NS} = \frac{c^{3/2} d \sqrt{(\nu F_\nu)} P_{NS}^2}{2\pi^{3/2} R_{NS}^{3/2} \sqrt{\eta}} = 2 \times 10^{13} d_{100\text{Mpc}} F_{30\text{Jy}}^{1/2} \tau_{5\text{msec}}^2 \sqrt{\nu_9} \eta_{-2}^{-1/2} \text{ G}. \quad (15)$$

The magnetic field (15) is somewhat larger than the typical  $\sim 10^{12} \text{ G}$  of young pulsars (recall that this estimate uses the FRB duration as an estimate of the spin period,  $B \propto \tau^2$ ), yet it is well within the overall distribution of rotationally-powered pulsars, especially given the uncertainties on other parameters. Also, if intrinsic duration of the FRB is much smaller than



the period, the estimate of the magnetic field (15) decreases accordingly. The corresponding spin-down time is

$$\tau_{SD} = \frac{\pi\eta I_{NS}}{d^2 F_\nu \mu P^2} \sim \text{few years.} \quad (16)$$

(The most important constraint in the above estimate comes from equating FRB duration with the rotation period of a neutron star. Duration of GPs is typically much shorter than the period.) Thus, *if FRBs are powered by the rotation of a neutron star it is required that (some fraction) of pulsars is born with millisecond periods (and normal magnetic fields).*

Observationally, the initial periods of neutron stars are generally unknown (the fastest young pulsar PSR J0537-6910 has 16 msec spin, Wang & Gotthelf 1998). The main way to probe initial spin periods of NSs is to obtain an independent age estimate (a SNR age, or a kinematic age, etc.) and to apply a usual magneto-dipole formula (with braking index three). The largest set of such calculations for NSs in SNRs has been presented by Popov & Turolla (2012). Despite, on average objects in this study appeared to have initial spin periods  $\sim 0.1$  s, significant fraction of analyzed sources can have very short initial periods. Even if pulsars are born with millisecond periods (but “regular” magnetic fields of  $\sim 10^{12}$  G) they are expected to quickly (within tens of years) spin down to periods larger than  $\sim 10$  msec (Lorimer et al. 1993; Lai et al. 2001; Vink 2008). Atoyan (1999) did argue in favor of fast initial spin in Crab. Theoretically, some simulations do predict rotation of neutron stars with the spins in the millisecond range (*e.g.* Camelio et al. 2016).

We conclude that a population of young pulsars (ages tens to hundreds of years) with magnetic fields typical of the observed population of young pulsars, but with spin periods in the few millisecond range is a viable source of FRBs (see also Cordes & Wasserman 2016; Connor et al. 2016b).

### 3.3. Frequency of occurrence

Let us do an estimate of the frequency of occurrence assuming that FRBs come from young powerful neutron stars in the local universe, from distances  $d \lesssim 100$  Mpc.

Dahlen et al. (2012) estimate the core-collapse SN rate  $\sim 3 \times 10^{-4} \text{ yr}^{-1} \text{ Mpc}^{-3}$ . Then in 100 Mpc we expect  $\sim 300$  SN per year (or one per day). If we assume that all young PSRs can produce strong bursts up to the age  $\sim 30$  yrs, then we have  $\sim 10^4$  such sources inside 100 Mpc. To have a rate of  $\text{few} \times 10^3$  FRBs per day, each pulsar needs to produce one-two bursts per day, roughly consistent with current overall limits (Petroff et al. 2015b). If we slightly increase the limiting distance (say, up to 200 Mpc), then we can obtain a more comfortable fraction ( $\sim 0.1$ ) of young PSRs having large  $\dot{E}$ , and so producing strong bursts.

One can estimate the rate of supergiant pulses from a given pulsar following the data given by Cordes & Wasserman (2016). The Crab pulsar produces GP with flux  $\sim 100 - 200$  kJy once per hour (the brightest one is slightly than an order of magnitude more luminous). If we take a pulsar with the same field but spin period  $\lesssim 2$  msec, then it has  $\dot{E} \sim 10^5$  times larger. So, we can expect from the same distance (2 kpc) flux  $(1 - 2) \times 10^{10}$  Jy. If we now consider distances 100-200 Mpc, then the flux is about few Jy. And the brightest — about few tens of Jy. Well in the range of FRB fluxes. Flux distribution for the Crab pulsar is roughly  $\propto S^{-3}$  (Cordes & Wasserman 2016). I.e., much brighter bursts (for example, like the Lorimer burst) might be rare – once in several months from the same source.

In the case of the repeating FRB 121102 (Spitler et al. 2016), the observed rate  $\sim 3$  hr $^{-1}$  is high, but potentially consistent with expectations for young PSRs since most bursts are of lower intensity, and some sources can be more active than average.

#### 4. Expected statistical properties of FRBs

Currently, only a handful of FRBs is known (Petroff et al. 2016). Let us now calculate statistical properties of FBRs expected in our model, which can be later tested with larger statistics. The key assumptions here is that the intrinsic luminosity of an FRB is proportional to the spin-down power  $\dot{E}$ .

##### 4.1. Injected and observed distribution in spin-down energy $f(\dot{E})$

Let's assume that pulsars are produced with a rate  $f_{inj}(\dot{E})$  (per unit time, per unit volume, per unit range of  $d\dot{E}$ ). The Boltzmann equation for the evolution of the number density  $f(\dot{E})$  reads

$$\partial_t f + \partial_{\dot{E}} \left( (\partial_t \dot{E}) f \right) = f_{inj}. \quad (17)$$

Assuming constant magnetic field (since we are interested in very young pulsars we neglect possible magnetic field decay), the spindown power evolves according to

$$\partial_t \dot{E} = - \frac{4B_{NS}R_{NS}^3}{c^{3/2}I_{NS}} \dot{E}^{3/2} \quad (18)$$

(the subscript  $NS$  refers to the surface properties of the neutron star).

The steady state kinetic equation in  $\dot{E}$  coordinates,

$$\partial_{\dot{E}} \left( \partial_t (\dot{E}) f(\dot{E}) \right) = f_{inj}(\dot{E}), \quad (19)$$

with the injection spectrum

$$f_{inj}(\dot{E}) \propto \dot{E}^{-\beta} \quad (20)$$

has a solution

$$f(\dot{E}) \propto c_1 \dot{E}^{-1/2-\beta} + c_2 \dot{E}^{-3/2}, \beta \neq 1 \quad (21)$$

$$f(\dot{E}) \propto \frac{\ln(\dot{E}_0/\dot{E})}{\dot{E}^{3/2}}, \beta = 1 \quad (22)$$

for the  $\beta \neq 1$  case the term with  $c_2$  is from the solution of the homogeneous equation; for the  $\beta = 1$  case  $\dot{E}_0$  is an integration constant.

## 4.2. Observed distribution in $\dot{E}$ : the inferred injection spectrum

As we discussed above, at high values of  $\dot{E}$  the observed and the injection spectra of pulsars are related by Eqs. (21-22). The special case of  $\beta = 1$  (same number of pulsars born per decade of  $\dot{E}$ ) is particularly interesting. Next, we demonstrate that the observed distribution of high spin-down power pulsars is indeed consistent with such fairly flat injection spectrum.

We use the ATNF catalogue (Manchester et al. 2005) to obtain the  $\dot{E}$  distribution. Since we are interested in young powerful pulsars, we use the high-energy tail of the distribution. We made several different radio pulsar samples from the ATNF catalogue to study the  $\dot{E}$  distribution. Two of them are shown in Fig. 1. Number distributions of pulsars per logarithmic bin is typically roughly  $\propto \log \dot{E}^{-0.5}$  which corresponds to  $dN/d(\dot{E}) \propto \dot{E}^{-1.5}$  (red dashed curve in left panel of Fig. 1). For some samples, for example the sample of PSRs with  $B > 10^{11}$  G,  $S_{1400} > 0.1$  Jy and distances  $> 7$  kpc (Fig. 1, right panel) a better fit is  $dN/d(\dot{E}) \propto \dot{E}^{-1.4}$ , still very close to the -3/2 law.

In addition, we analyzed period distribution of short period pulsars. Observed distribution in  $P$  for short period pulsar (from  $\sim 0.03$  to  $\sim 0.2$  s) is  $f(P) \propto P^{1/2}$ . Though this range of periods is also populated by older objects, it contains many young sources, in correspondence with estimates by Popov & Turolla (2012). Thus, it can be used to estimate the initial  $\dot{E}$  distribution.

Since for the simple magneto-dipole formula  $P \propto \dot{E}^{-1/4}$  this translates to

$$f(\dot{E}) \propto \dot{E}^{-11/8}, \quad (23)$$

This is sufficiently close to the  $\alpha = -3/2$  law. Thus, we conclude that the observed distribution of fast pulsars is consistent with injection parameters  $\beta = 1$ ,  $f_{inj} \propto 1/\dot{E}$  (equal

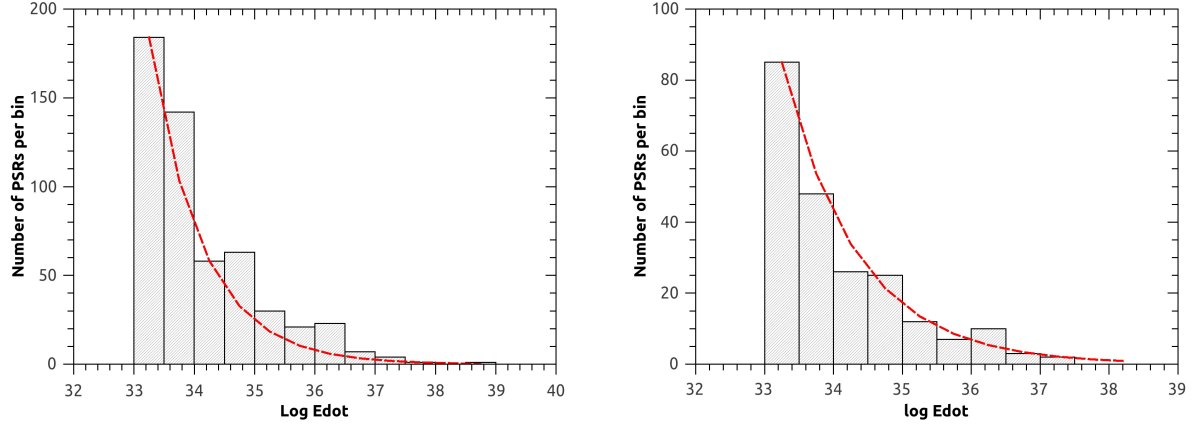


Fig. 1.— *Left Panel.* Differential  $\dot{E}$  distribution above  $\dot{E} = 10^{33} \text{ erg s}^{-1}$  in log-scale. Selected PSRs have  $B > 10^{11} \text{ G}$ ,  $S_{1400} > 0.1 \text{ Jy}$ . Dashed line corresponds to the law  $dN/d(\dot{E}) \propto \dot{E}^{-1.5}$ . *Right Panel.* Differential  $\dot{E}$  distribution above  $\dot{E} = 10^{33} \text{ erg s}^{-1}$  in log-scale. Selected PSRs have  $B > 10^{11} \text{ G}$ ,  $S_{1400} > 0.1 \text{ Jy}$ , and distances  $> 7 \text{ kpc}$ . Dashed line corresponds to the law  $dN/d(\dot{E}) \propto \dot{E}^{-1.4}$ .

number of newborn sources per decade of  $\dot{E}$ ). The distribution in spin-down power  $f(\dot{E})$  can be related to the multivariate distribution in period and magnetic field  $f(B, P)$ , see §A.

### 4.3. Homogeneous source distribution

We analyzed the observed  $\text{Log } N - \text{Log } S_{\text{peak}}$  distribution of FRBs, Fig. 2, using the FRB catalogue (Petroff et al. 2016). If we exclude the brightest burst, the Lorimer burst, the distribution is compatible with the isotropic  $-3/2$  law, Fig. 2. In addition, the distribution in  $F_{\text{obs}}$  for 13 dimmest sources has a very peculiar form: it is linear in the linear scale. However, statistics is low. Deviations from the  $3/2$  law are expected in radio surveys, since the effective area of the telescope beam is a strong function of flux - super-bright sources (like the Lorimer burst) can be found further away from the centre of the radio beam. This biases the  $\text{Log } N - \text{Log } S$  towards flatter apparent spectral distributions (Li et al. 2016, also found flatter distribution). In addition, low statistics seems to bias the  $\text{Log } N - \text{Log } S$  distribution towards flatter indices. We have performed a number of trials selecting 16 sources from various luminosity distributions and fitting with the power-law. We notice that, first, for the small number of sources the average value of the power-law index was below  $3/2$  and, second, the standard deviation for 16 sources was  $\sigma \approx 0.2$ . We conclude that the

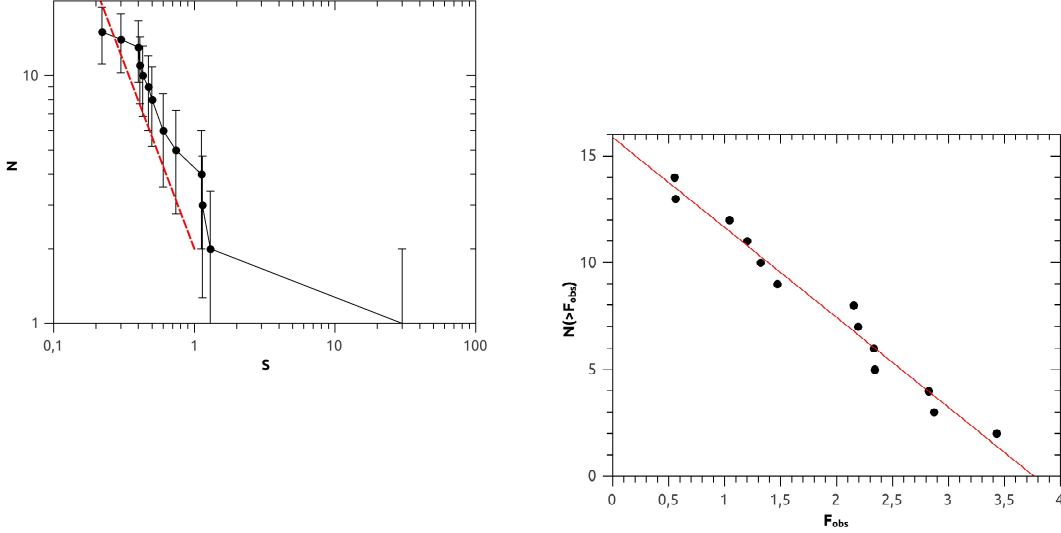


Fig. 2.— *Left Panel.* Log  $N$  – Log  $S_{\text{peak}}$  distribution for FRBs. The observed distribution is consistent with homogeneous distribution of sources. The brightest burst, the Lorimer burst, is excluded from the fit. *Right Panel.* Distribution  $N(> F_{\text{obs}})$ – $F_{\text{obs}}$  on the linear-linear scale. Two brightest sources are not included.

observed distribution is consistent with homogeneous source distribution.

#### 4.4. DM-peak flux correlation

Combining expressions for DM (7) with spin-down power ( $\dot{E}_0$  is the value at birth,  $\tau$  is the initial spin-down time)

$$\dot{E} = \frac{\dot{E}_0}{(1 + t/\tau)^2}, \quad (24)$$

we find

$$\text{DM} = \frac{M_{ej}^2}{2E_{ej}m_p\tau} \frac{\dot{E}}{(\dot{E} + \dot{E}_0)^2} \quad (25)$$

(recall that we use  $\dot{E}$  as a proxy for peak luminosity). Thus, for times  $t \ll \tau$ , when  $\dot{E} \approx \dot{E}_0$  we expect that DM is independent of the  $\dot{E}$  and, under assumptions of the model, of  $S_{\text{peak}}$ .

For longer times, DM should decrease with  $\dot{E}$  (and  $S_{peak}$ ),  $DM \propto \dot{E}$ .

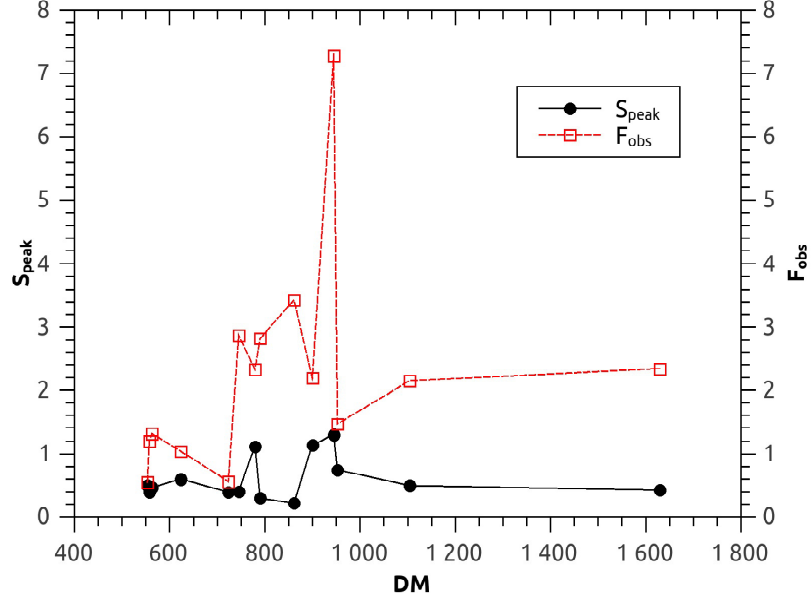


Fig. 3.— Peak luminosity,  $S_{peak}$ , and fluence,  $F_{obs}$  vs. dispersion measure (from the FRB catalogue Petroff et al. 2016). The Lorimer burst (FRB 010724) is removed from the plot as it is too bright in comparison with the others.

In Fig. 3 we plot the observed peak luminosity,  $S_{peak}$ , and fluence,  $F_{obs}$ , vs. dispersion measure. Obviously, there is no strong dependence of  $S_{peak}$  and  $F_{obs}$  on DM. By itself, this behavior of DM excludes models in which dispersion measure is a proxy of distance (as dispersion happens in the extragalactic medium) and bursts are more or less standard candles.

#### 4.5. Logarithmic injection in $\dot{E}$ ( $\beta = 1$ ): implications for the radial distribution of brightest sources

The spin-down power distribution  $f(\dot{E}) \propto \dot{E}^{-3/2}$  is, in many respects, a special case: the dipole spin-down law (with constant magnetic field) singles out this solution as a special one (this is a solution of a homogeneous Boltzmann equation for the pulsar flow); also, it is consistent with the observed spin-down distribution of fast pulsars (see §4.2). The observed spectrum 3/2 implies, approximately, a special injection spectrum  $\beta = 1$  – equal number of

newborn pulsars per decade of  $\dot{E}$ . Next we calculate the expected observed properties of FBRs for this special injection spectrum  $\beta = 1$  (again, assuming that intrinsic brightness correlates with spin-down luminosity).

First, we evaluate the expected distribution of distances for a given observed flux. In an unlimited volume the distribution of fluxes follows the  $N(> S) \propto S^{-3/2}$  law independent of the intrinsic luminosity distribution. On the other hand, the distribution of sources contributing a given flux in distances depends on the intrinsic luminosity function. The injection spectrum  $f_{inj} \propto 1/\dot{E}$  translates into steady state observed distribution (22)  $f(\dot{E}) \propto \ln(\dot{E}_0/\dot{E})/\dot{E}^{3/2}$ . Neglecting for a moment slowly varying logarithm, the case  $\alpha = 3/2$  turns out to be an interesting special case: the distance to the nearest source is  $r \sim f(\dot{E})^{-1/3}$  and the observed flux (again, assuming that FRB luminosity follows  $\dot{E}$ )

$$S \propto \frac{\dot{E}}{r^2} \propto \dot{E} f(\dot{E})^{2/3} \quad (26)$$

For  $f(\dot{E}) \propto \dot{E}^{-\alpha}$  this implies

$$S \propto \dot{E}^{(1-2\alpha/3)} \propto r^{-2+3/\alpha} \quad (27)$$

So, for  $\alpha < 3/2$ ,  $S$  increases with  $r$  – the brightest sources are far away. For the special case  $\alpha = 3/2$ , the observed brightness is independent of the distances,  $S \propto r^0 \propto \dot{E}^0$  (in a larger volume there are brighter sources – this a variant of a so-called Malmqvist bias).

Thus, we expect that for the logarithmic injection spectrum,  $\beta = 1$ , at a given flux the observed sources are distributed over a wide range of distances. On a more subtle point, for the logarithmic injection spectrum the expected  $\dot{E}$  distribution (22) differs slightly from  $3/2$ , by a logarithm; thus we still expect that closer sources are brighter, yet the brightest sources are distributed over a large distance. This is confirmed by our Monte Carlo simulations which we discuss next.

We conclude that for the injection spectrum  $f_{inj} \propto 1/\dot{E}$  the observed brightest sources have very broad spacial distribution - the brightest one hundred sources (out of approximately a million in the total sample) are located within  $\sim$  a quarter of the test volume. This is important: isotropy of FRBs imply that the brightest ones cannot come from nearby sources — the local Universe is highly inhomogeneous on scales of tens of Mpc.

#### 4.6. Monte-Carlo simulations of pulsars' spin-down and observed brightness distribution

To test the spatial distribution of the brightest sources we have conducted simulations of pulsar population. First, to test the spacial distribution of brightest sources we injected

pulsars with the expected steady state distribution (22) over a range of distances. The results are shown in Fig. 4. Importantly, this confirms that brightest FRBs come from a wide range of distances.

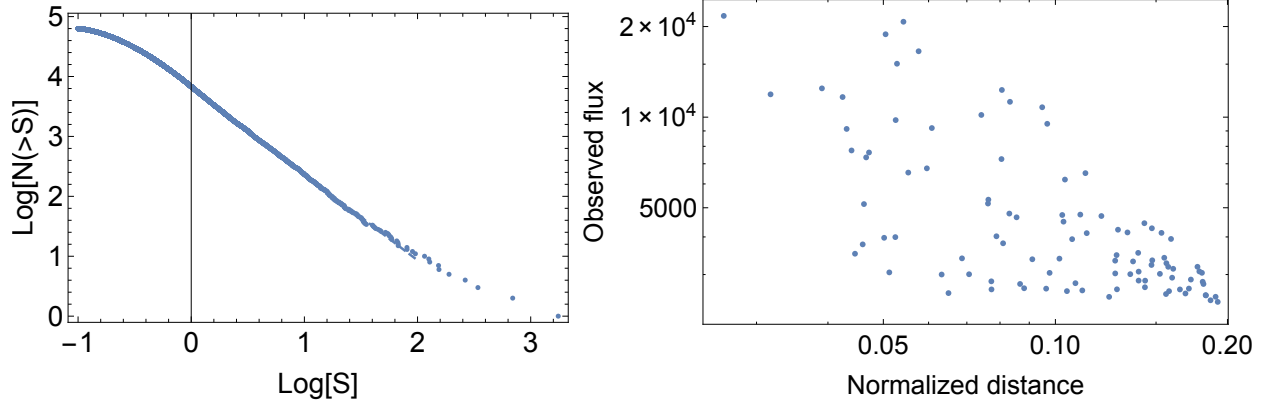


Fig. 4.— *Left Panel.* Observed distribution of fluxes  $\propto \dot{E}/r^2$  for injection spectrum (22). The brightest source are well fitted with  $-3/2$  spectrum (dashed line, fitted slope 1.53). *Right panel.* Radial distribution of the hundred brightest sources (out of total number of approximately one million, located at normalized distances between 0 and 1). This plot shows that the brightest sources are, generally, located in a broad range of distances.

Second, we did Monte Carlo runs injecting neutron stars by supernovae and following subsequent spin down evolution. At each time step we redistribute homogeneously a number of neutron stars in a volume  $0 < r < 1$  with initial distribution  $f_{inj} \propto \dot{E}^{-1}$ ,  $0.1 < \dot{E} < 1$ . Neutron stars spin down according to the magneto-dipole formula  $\partial_t \dot{E} \propto -\dot{E}^{-3/2}$ . The observed FRB flux is parametrized with spin-down luminosity,  $S \propto \dot{E}/r^2$ . Sources with flux below some minimal value are discarded. After sufficiently large number of time steps the total number of pulsars reaches statistical equilibrium. The distribution function  $f(\dot{E})$  approaches the limit (22), Fig. 5, left panel, while the distribution of brightness approaches  $-3/2$  power law, Fig. 5, right panel

In conclusion, our MC simulations confirm the analytical estimates: the spindown distribution follows (22), the brightest observed sources are distributed over a wide range of distances, and, naturally, that the expected sources count follows  $N(> S) \propto S^{-3/2}$ .



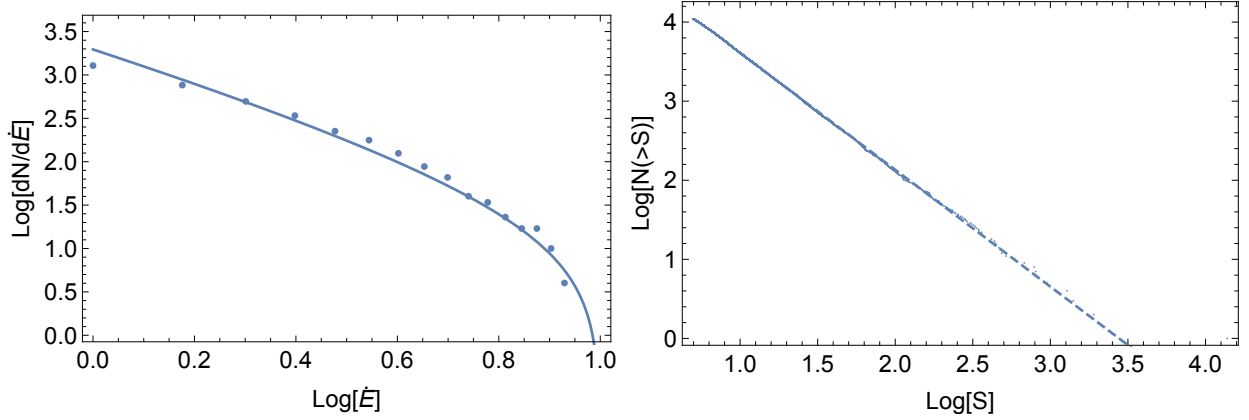


Fig. 5.— *Left Panel.* Comparison of MC calculations with spin-down and the expected analytical distribution (22). Slight disagreement is probably due to “edge effects” (small dynamical range). *Right panel.* Observed distribution of fluxes  $\propto \dot{E}/r^2$  for MC simulations. The high-S tail is fit with power-law 1.47.

## 5. Predictions

In this paper we argued that the physical constraints imposed by the properties of FRBs limit their origin to the magnetospheres of neutron stars. Two special types could satisfy those constraints: fast rotating young neutron stars (using the rotational energy to generate FRBs), or very high magnetic fields neutron stars — magnetars (using the magnetic energy). The key distinction between the two possibilities would be a detection of high energy emission contemporaneous with an FRB — Crab giant pulses do not show high energy signals (Bilous et al. 2012; Mickaliger et al. 2012; Aliu et al. 2012).

In this paper we discussed possible observational features of the GP-FRB association. (i) Since we associate FRBs with recent core-collapse explosions, we expect that a SN might have been detected years before the burst. We encourage observers to search in archives for such correlations. (A possible exception to this could be alternative channels of neutron star formation, like accretion-induced collapse (Nomoto & Kondo 1991); such events should have low DMs. Magnetar activity is expected to be delayed from the formation of a neutron star by corresponding Hall time, which can vary from years to millennia depending on the location within the crust (Lyutikov 2015).) (ii) As we expect that distances are  $\lesssim 100 - 200$  Mpc, then it might be possible to identify the host galaxy, which will have significant star formation rate. (iii) We expect the repetition rate of FRBs of the order of one per day per source. (iv) For a given FRB source the DM through a newly ejected SNR should decrease with time (the repeating FRB 121102 did not show such a predicted behavior —

possible mitigating factors could be: large Galactic contribution, at least 30% but possibly higher; older SN, few hundred years; possibly large contribution from the host galaxy (as opposed to the surrounding SNR). At the same time the observed brightness might either be independent of time and of the DM (if the observation time after a SN is shorter than the initial spin-down time), or to decrease with time (if the observation time after a SN is longer than the initial spin-down time). (v) True distances to FRB sources will show large variations (not necessarily the closer – the brighter). (vi) Some pulsar are born with very fast spins, of the order of few milliseconds. Most of the above predictions assume scaling of intrinsic luminosity with the spin-down power.

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### A. $f(\dot{E})$ distribution from $f(B, P)$

Let’s assume that at birth the distribution of magnetic fields and periods is  $f(B, P)$ . We can parametrize the spin-down power as

$$\dot{E} = \dot{E}_0 \left( \frac{B}{B_0} \right)^2 \left( \frac{P}{P_0} \right)^{-4} \quad (\text{A1})$$

where  $B_0$ ,  $P_0$  and  $\dot{E}_0$  are some fiducial values of the magnetic field, period and spin-down power. For a given  $\dot{E}$  we have

$$\begin{aligned} B/B_0 &= (\Phi/2)(P/P_0)^2 \\ \Phi &= 2\sqrt{\dot{E}/\dot{E}_0} \end{aligned} \quad (\text{A2})$$

(function  $\Phi$  is proportional to the total electric potential).

We can introduce a coordinate  $\Psi$  orthogonal to the electric potential

$$\begin{aligned} \Psi &= \frac{1}{4} \left( 2 \left( \frac{B}{B_0} \right)^2 + \left( \frac{P}{P_0} \right)^2 \right) \\ P/P_0 &= \sqrt{\frac{\sqrt{8\Phi^2\Psi + 1} - 1}{\Phi^2}} \\ B/B_0 &= \frac{\sqrt{8\Phi^2\Psi + 1} - 1}{2\Phi}, \end{aligned} \quad (\text{A3})$$

see Fig. 6. (If magnetic field remains constant and  $P_0$  and  $B_0$  are the initial values, a given pulsar follows a line  $\Psi = (1 + \Phi)/(2\Phi)$  starting from a point  $\Phi_0 = 2$  and  $\Psi_0 = 3/4$ .)

The Jacobian of the transformation  $\{B, P\} \rightarrow \{\Phi, \Psi\}$  is

$$J = -\sqrt{\frac{\sqrt{1 + 8\Phi^2\Psi} - 1}{\Phi^2(1 + 8\Phi^2\Psi)}} \quad (\text{A4})$$

To find distribution in potential (and spin-down power)  $f(\Phi)d\Phi = f(\sqrt{\dot{E}})\dot{E}^{-1/2}d\dot{E}/2$  we need to integrate

$$f(\Phi, \Psi) = f(B, T)J \quad (\text{A5})$$

(where magnetic field and period are expressed by Eq. (A3) ) over  $\Psi$  (from zero to infinity).

For example, for a log-normal injection distribution in both magnetic field and period, with mean  $P_0$  and  $B_0$  and dispersions  $\sigma_{P,B}$ , the resulting  $\dot{E}$  distribution is also log-normal.

$$\begin{aligned} f(\dot{E}) &= \frac{e^{-\ln^2(\dot{E}/\dot{E}_0)/2\sigma^2}}{2\sqrt{2\pi}\sigma} \frac{1}{\dot{E}} \\ \sigma &= \sqrt{\sigma_B^2 + 4\sigma_P^2} \\ \bar{\dot{E}} &= e^{2\sigma^2} \dot{E}_0 \end{aligned} \quad (\text{A6})$$

Log-normal distribution closely resembles the  $1/\dot{E}$  power-law over a broad range of parameters.



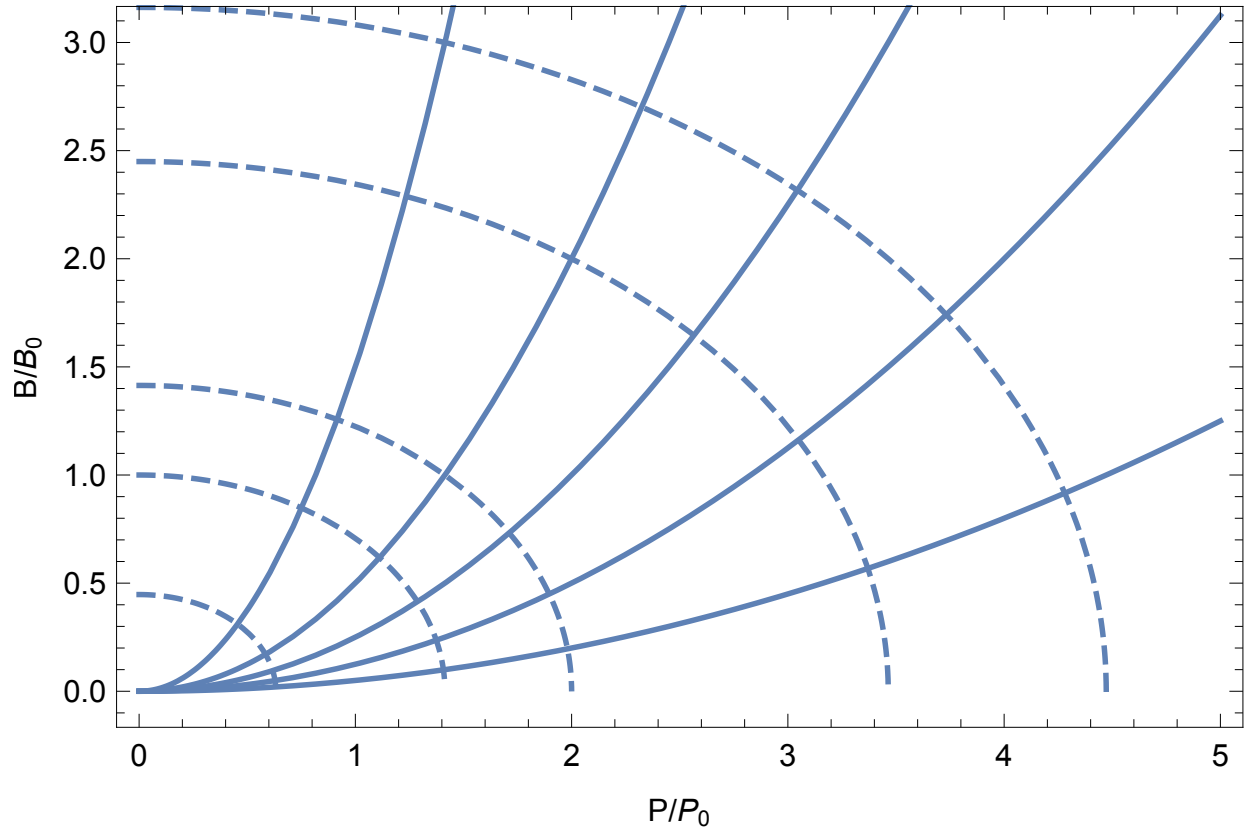


Fig. 6.— Lines of constant  $\sqrt{E} \equiv \Phi$  (solid) and orthogonal curves (lines of constant  $\Psi$ ).